Technical Notes

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Electromagnetic Levitation with Acoustic Modulation for Property Measurement

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Nomenclature

a = characteristic length of levitated material, m
$\boldsymbol{B}, B$ = magnetic flux density, Wb/m^2
c_p = specific heat of levitated material (J/kg K)
E = ohmic heating rate, W
$\boldsymbol{g}, g$ = gravitational acceleration
I = moment of inertia, $I = 2/5\ ma^2$
$\boldsymbol{J}, J$ = induced eddy current density (A/m^2), $\boldsymbol{J} = 1/\mu \nabla \times \boldsymbol{B}$
k = wave number of sound waves in the medium
K = radius of curvature of deformed droplet, m
m = mass of droplet, kg
P = pressure, N/m^2
P_m = mean magnetic pressure on the droplet surface, N/m^2
P_s = pressure amplitude of acoustic standing wave
P_{20} = quadrupole radial projection of acoustic radiation pressure
$\bar{P}$ = nondimensional acoustic pressure ratio, $P_{s,1}/P_{s,\mathrm{ref}}$
$\bar{P}_{\mathrm{dr}}$ = acoustic driving pressure ratio
$\bar{P}_{\mathrm{lev}}$ = acoustic levitating pressure ratio
q_j = ohmic heating rate, $\approx E/a^2$
r = radius of deformed droplet, m
R = equilibrium (spherical) radius of the droplet, m
t = time, s
T = torque due to electromagnetic forces, Nm
T_m = melting point temperature, K
T_0, T_{ref} = reference (room) temperature, K
V = voltage applied at acoustic transducer, V
x = deformation of droplet
β, β^* = adiabatic compressibility of the medium and sample, respectively
γ = surface tension of levitated material, N/m
δ = skin depth of penetration of magnetic field, m, $1/\sqrt{(1/2)\omega\mu\sigma}$
λ = magnetic diffusivity, m^2/s, $1/(\mu\sigma)$
μ = magnetic permeability of the medium, $4\pi \times 10^{-7}$
ρ = density of the medium, kg/m^3
ρ^* = density of the levitated material, kg/m^3
σ = electrical conductivity of the levitated material, $\Omega^{-1}m^{-1}$
ω = angular frequency
ω_r = angular frequency of rotation

Subscripts
1 = material or sample being considered
ref = reference material (water)

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Introduction

MOST materials at high temperatures are highly corrosive and present the problem of crucible contamination. To overcome this problem, many levitation/melting techniques have been developed, making the prospect of containerless processing of materials realizable, especially with reference to their applications in the reduced gravity environment of space.[1]

Electromagnetic techniques are used to levitate and melt metal specimens. The levitated liquid droplets also exhibit shape oscillations that are used to obtain thermophysical properties. However, an electromagnetic levitator does not possess the manipulative capabilities that are associated with acoustic methods, and cannot levitate poor conductors. Acoustic levitation techniques are versatile with regard to the variety of samples that can be studied, although most of the applications have been at low temperatures and under isothermal conditions.[2,3]

A suitable combination of electromagnetic and acoustic methods has been suggested[4,5] as a more reliable means for high-temperature thermophysical property measurements. For example, such a combination may employ high-intensity acoustic forces to drive shape oscillations of an electromagnetically levitated droplet. We present here a comparative analysis of the requirements of acoustic and electromagnetic systems and, using existing theory, assess the feasibility and the acoustic forces necessary to drive electromagnetically levitated and melted materials, both in the presence of terrestrial gravity and microgravity.

Analysis

Electromagnetic Forces

A specimen placed in an alternating electromagnetic field experiences Lorentz forces and ohmic heating caused by the induced currents. When these are of a sufficient magnitude, the specimen may be supported against gravity and may be heated until it melts. From classical electromagnetic theory, the magnetic flux $\boldsymbol{B}$ for an axisymmetric current distribution is

$$\frac{\partial \boldsymbol{B}}{\partial t} = \lambda \nabla^2 \boldsymbol{B} \tag{1}$$

For levitation to occur, the time-averaged force must exceed the gravitational force on the specimen. This condition implies that

$$\int_V [\tfrac{1}{2}\ Re\ (\boldsymbol{J} \times \boldsymbol{B}) + \rho^* \boldsymbol{g}]\ \mathrm{d}V = 0 \tag{2}$$

As the skin depth δ becomes much smaller than the characteristic length a of the droplet, it is essentially supported by the mean magnetic pressure P_m over the surface and experiences an ohmic heating E caused by induced currents:

$$P_m = \frac{1}{4\mu} B^2 \tag{3}$$

$$E = \int_V J^2 \mu\lambda\ \mathrm{d}V \tag{4}$$

Table 1 Properties of some liquid metals at or near their melting points

Material	T_m, K	ρ, Kg m^{-3}	σ, $\Omega^{-1}m^{-1} \times 10^{-6}$	γ, N m^{-1}	c_p, J kg^{-1} K^{-1}
Copper[7]	1356	8240	4.74	0.135	494
Aluminum[7]	933	2370	5	0.9156	1084
Gallium[7]	302.8	6100	6.1	0.735	408
Nickel[9,10]	1726	7705	1.18	1.778	670

Equations (1-4) have been solved analytically[6,7] and numerically[8] in studying the fluid dynamical aspects of levitation melting. In this work, further analysis of the electromagnetic problem will be performed in an order-of-magnitude sense, as this will be sufficient for estimating the acoustic force requirements.

For successful electromagnetic levitation, Eq. (2) implies that

$$P_m a^2 \approx mg$$

or, from Eq. (3),

$$B \approx (\rho^* \mu g a)^{1/2} \tag{5}$$

To order of magnitude, Eq. (4) now implies that

$$E \approx \frac{B^2}{\mu} a^2 \lambda^{1/2} \omega^{1/2} \tag{6}$$

and the time t_m required to raise the specimen from a base temperature T_0 to the melting temperature T_m is of the order

$$t_m \approx [\rho^* a c_p (T_m - T_0)]/q_j \tag{7}$$

The reported frequency of rotation caused by electromagnetic forces is between 30-50 Hz.[5] The torque T for the solid body rotation of the droplet is

$$T = I \omega_r^2 \tag{8}$$

For a given material, the levitating force is of the order of its weight. Then, using its properties and Eqs. (5-7), order-of-magnitude estimates can be made for any given material. In this work, we consider the metals listed in Table 1.

Acoustic Forces

Marston[11] has developed a linear inviscid theory to describe the acoustic radiation pressure on a compressible sphere and evaluated the quadrupole projection as

$$P_{20} = [-3(5\pi)^{1/2}/20]\, P_s^2 \beta [1 + 7/5(kR)^2] + O(kR)^4 \cdots \tag{9}$$

Assuming small deviations of the interface from an original spherical form, it is convenient to write the radius of the deformed drop $r(\theta)$ in the form

$$r(\theta) = R + x(\theta)$$

where $x(\theta)$ can be expanded in spherical harmonics and reduces to

$$x(\theta) = (5/16\pi)^{1/2}(3 \cos^2\theta + 1)(R^2/4\gamma) P_{20} \tag{10a}$$

$$x(\theta) \sim \frac{P_s^2 \beta R^2 [1 + 7/5(kR)^2]}{\gamma} \tag{10b}$$

In practice, the standing acoustic wave (carrier wave) is modulated by a low-frequency signal that drives shape oscillations of the droplet. The amplitude of the response to a modulated carrier wave is proportional to the square of the unmodulated carrier pressure amplitude. Thus, if a particular carrier pressure $P_{s,\mathrm{ref}}$ causes a deformation x_{ref} and, when modulated, drives shape oscillations of a detectable amplitude for a reference material, then a pressure amplitude $P_{s,1}$, which causes a comparable deformation, can drive, when modulated, similarly detectable shape oscillations in a material 1.

$$\frac{x_1(\theta)}{x_{\mathrm{ref}}(\theta)} \approx 1 \approx \frac{P_{s,1}^2}{P_{s,\mathrm{ref}}^2} \frac{\gamma_{\mathrm{ref}}}{\gamma_1} \frac{\beta_1^*}{\beta_{ref}^*} \tag{11}$$

We now address the levitation forces. Apfel[2] obtained the vertical force balance for the case of an arbitrary standing wave field under the assumption that the sample had a size much smaller than the acoustic wavelength. This can be stated as

$$P(z)\, dP(z)/dz = -(2\rho g/\beta)(1 - \rho^*/\rho)/[\beta^*/\beta - (5\rho^* - 2\rho)/(2\rho^* + \rho)] \tag{12}$$

For liquids or solids levitated in a gaseous medium, $\beta^*/\beta \ll 1$, and $\rho^*/\rho \gg 1$. We also assume that the pressure $P(z)$ and the gradient $dP(z)/dz$ are directly proportional to the input voltage V at the transducer and, hence, the pressure amplitude of the standing wave P_s. Then Eq. (12) becomes

$$V^2 \sim P_s^2 \sim \frac{\rho^*}{\beta^*} g \tag{13}$$

From Eq. (13) we have

$$\frac{V_1}{V_{\mathrm{ref}}} = \frac{P_{s,1}}{P_{s,\mathrm{ref}}} = \bar{P} = \sqrt{\left(\frac{\rho_1^*}{\rho_{\mathrm{ref}}^*} \frac{\beta_1^*}{\beta_{\mathrm{ref}}^*}\right)} \tag{14}$$

Using Eqs. (11) and (14), we can obtain values of the driving pressure ratio $\bar{P}_{\mathrm{dr}}$ and the levitating pressure ratio $\bar{P}_{\mathrm{lev}}$ for any material in comparison to a reference.

Results and Discussion

Results are given for four metals: Cu, Al, Ga, and Ni. Table 1 lists the relevant properties for these metals at or near their melting point. The characteristic length of the droplets was taken to be $\approx 10^{-2}$ m, and the weight to be $\approx 10^{-3}$ kg. This covers most of the practical cases where about 1 g of a sample of radius about 5-30 mm is levitated electromagnetically. The results for both the electromagnetic and the acoustic part for Earth's gravity are listed in Table 2. As can be seen, the electromagnetic torque is of the order of 10^{-3} Nm and can be countered by a force of the same order as the levitating force, applied at the moment arm a. The heating time t_m is very small in all cases, much smaller than in the case of resistively heated furnaces (acoustic levitator furnaces, for example, have a 30 min heating time for a 1500°C rise in temperature). The heating rate for the case of copper as an example is of the order of 10^5 W/m^2. This suggests that although high frequencies and currents are required to levitate a sample, much lower frequencies and currents may suffice to melt it satisfactorily. In microgravity such a situation is advantageous in terms of thermal efficiency.

Two cases are considered for β in making acoustic calculations: 1) the medium is at the reference temperature, and 2) the medium is at the temperature of the levitated material. Case 1 is equivalent to assuming that radiation is the dominant

Table 2 Requirements of electromagnetic and acoustic methods on Earth for some liquid metals

Metal	B, Wb m^{-2}	$q_j \times 10^{-5}$, Wm^{-2}	t_m, s	$T \times 10^{-3}$, Nm	$\bar{P}_{lev}$, Case 1	$\bar{P}_{dr}$, Case 1	$\bar{P}_{lev}$, Case 2	$\bar{P}_{dr}$, Case 2
Copper	0.03	2.4	179	3	3	1.5	6.4	3.2
Aluminum	0.02	1.0	162	3	1.5	3.6	2.7	6.4
Callium	0.03	2.1	~0.5	3	2.5	3.2	2.5	3.2
Nickel	0.03	4.67	157	3	3	5	7.2	12

Table 3 Requirements of electromagnetic and acoustic methods in microgravity for some liquid metals

Metal	$B \times 10^{-5}$, Wb m^{-2}	q_j, W m^{-2}	t_m, s	$\bar{P}_{lev} \times 10^3$, Case 1	$\bar{P}_{lev} \times 10^3$, Case 2
Copper	3	0.23	2×10^8	3	6.4
Aluminum	2	0.1	1.6×10^8	1.5	2.7
Gallium	3	0.21	3.3×10^5	2.5	2.5
Nickel	3	0.47	1.5×10^7	3	7.2

mode of heat transfer and the medium is radiatively nonparticipating. Case 2 is equivalent to an isothermal furnace. In all calculations a sound pressure level (SPL) of 160 dB, required to levitate a droplet of water in air at room temperature,[12] is used as reference. The acoustic forces for liquid metals can be as much as 12 times greater than that for water, as shown in Table 2.

For the combined acoustic-electromagnetic case, the interface condition is

$$\gamma K + P_m + P_s = 0 \tag{15}$$

Thus, for the combined case, in the most conservative estimate, Eq. (15) implies that the effective pressure at the interface is $(P_s - P_m)$. The application of this result is explained for the case of copper (row 1 of Table 2). Since P_m is of the order of the levitating pressure, and $P_s \approx 3P_{s,\text{ref}}$ is the acoustic levitating pressure, it follows that $P_m \approx 3P_{s,\text{ref}}$. The acoustic driving pressure now becomes, from Eq. (15), $4.5P_{s,\text{ref}}$ in order to keep the effective interface pressure at $1.5P_{s,\text{ref}}$. Thus, to order of magnitude, the acoustic driving force in the presence of an electromagnetic field is just a sum of the driving and levitating forces for the purely acoustic cases.

Table 3 is a list of some of the requirements for systems in microgravity. The driving forces do not change since they are independent of gravitational acceleration (and are dependent on the surface tension). The levitating forces, as Table 3 reveals, are at least three orders of magnitude lower than in Earth's gravity. The heating time is very large for a minimum B. This suggests that it may be necessary to increase either B, or the frequency, to obtain better heating.

The analysis of this work indicates that a combination of the electromagnetic and acoustic forces is feasible. In Earth-based experiments, the requirements are higher than in the low-temperature acoustic applications. In microgravity, however, the results are highly encouraging, since very minimal forces are required, and the equilibrium shapes are more nearly spherical. In addition, radiation is the dominant mode of heat transfer under microgravity conditions, and since the inert atmospheres usually used in these applications do not participate significantly, the assumption of negligible temperature gradient is less restrictive.

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CO_2 Laser Absorption in SF_6-Air Boundary Layers

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Introduction

GASEOUS sulfur hexafluoride (SF_6), which absorbs radiation at 10.6 μm, can be injected into boundary

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